

Definition:

The distance of X, denoted $\text{Dist}(X)$, is the number of pieces between X and X-1 going counter-clockwise around the loop. For example, $\text{Dist}(2)$ is the number of pieces between 2 and 1.

Example:

Given the following configurations:
4, 5, 10, 2, 19, 20, 23, 15, 18, 1,

Where reading left to right is considered counter-clockwise around the loop.
Then $\text{Dist}(2) = 5$;

Note: When the game is solved $\text{Dist}(Y) = 0$ for all pieces Y.

Definition:

Let R denoted the permutation that moves each piece counter-clockwise one piece. S will denote a spin move. In cycle notation:

$$R = (1, 2, 3, 4, 5, \dots, 19, 20)$$

$$S = (1, 4)(2, 3)$$

Reachable States

$$\text{Let } X = [\text{SRSR}^{-1}\text{SRSR}^3]$$

$$Y = X^5$$

$$Z = YR^{-1}$$

In cycle notation $Z = (1, 2)$, that is it interchanges 1 and 2 while fixing every other piece.

By placing any piece in the left most positions of the spinner and applying Z we interchange that piece with the piece directly to its right. Thus every configuration is reachable from the initial configuration by applying this method.

AlgorithmPlacing pieces 1-16:

Let $X = 2$, i.e. begin by placing piece 2;

- While X is not piece 16 do:
 - While $\text{Dist}(X)$ is greater than 3
 - Apply R until an application of S permutes X.
NOTE: This will place X in the right-most position of the spinner
 - Apply S

- Apply R^{-3}
 - Determine $\text{Dist}(X)$
- Now $\text{Dist}(X)$ is less than or equal to 3
- Place X in the left most position of the spinner
- If $\text{Dist}(X) = 3$
 - Apply $R^{-3}S$
- Else, if $\text{Dist}(X) = 2$
 - Apply $R^{-1}SR^{-1}S$
- Else, if $\text{Dist}(X) = 1$
 - Apply $R^{-1}SRSR^{-1}S$
- $X = X + 1$ (we now look at the next piece and repeat these steps)

Placing pieces 17-20:

Recall the procedures $X = [SRSR^{-1}SRSR^3]$
 $Y = X^5$

Let $X = 17$, that is X is piece 17

- While the puzzle is not solved
 - While $\text{Dist}(X) > 0$
 - Place the piece directly to the left of X in the left most position of the spinner.
 - Apply Y
 - $X = X + 1$ (we now consider the next piece)